The 3rd Mission Idea Contest for Micro/Nano-satellite Utilization

Lunar Relativistic Positioning System for human exploration

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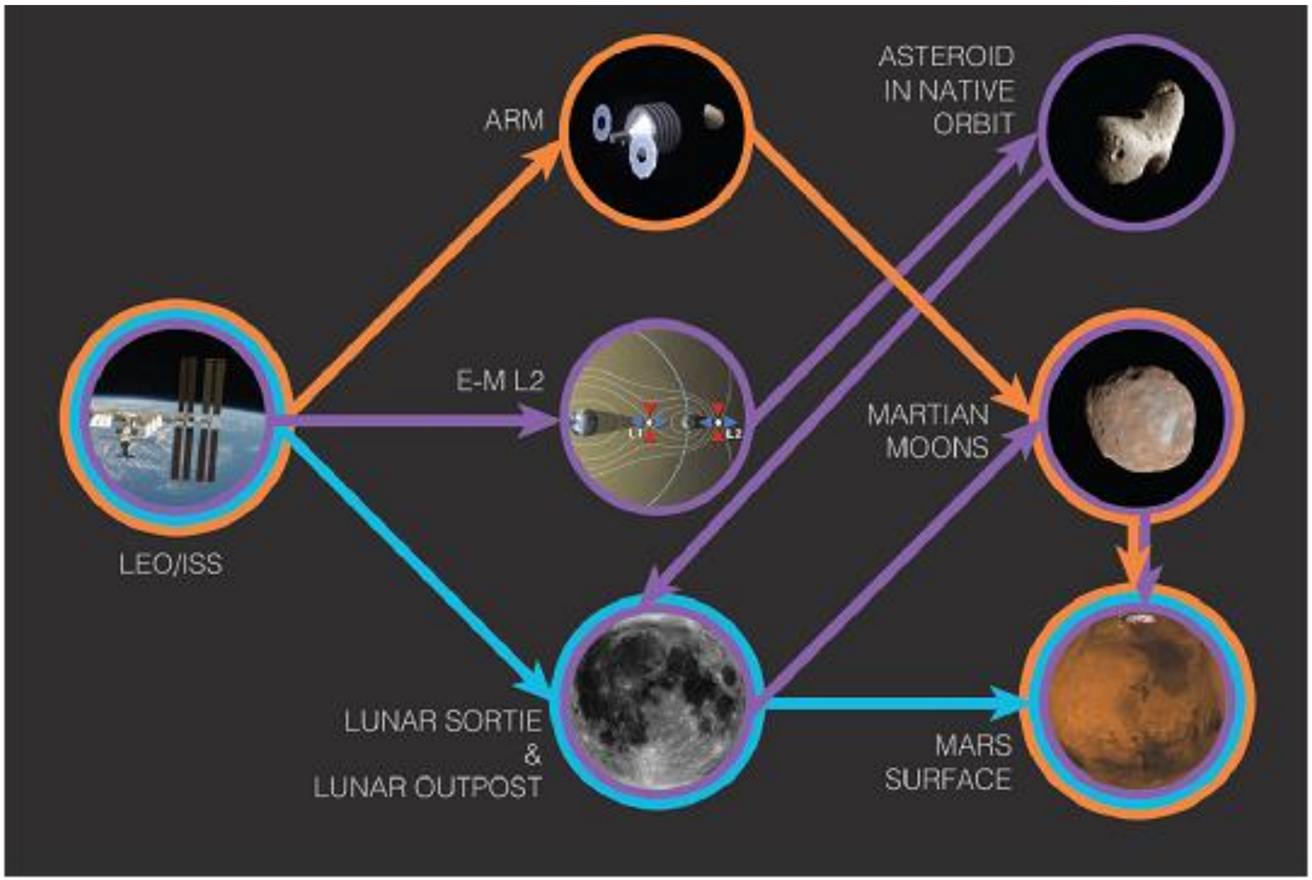
Introduction
Concept of Operations
Space Segment
Ground Segment
Implementation
Conclusions

Mission Need

Future of human space exploration:



Mission Need

Future of human space exploration:

- Moon, Mars, …
- Permanent outposts (e.g. lunar South Pole)
- Planetary exploration: why?
  - ✓ Scientific activities (samples, water ice, minerals, …)
  - ✓ In-situ resource utilization (outpost sustainment)
- Planetary exploration: how?
  - ✓ Around the base
  - ✓ Far from the base
- Rovers
  - ✓ Manned and unmanned
  - ✓ Pressurized and not pressurized
- **Positioning/Navigation** becomes a major problem
  - ✓ Need for a navigation system, like GPS, GLONASS, Galileo, Beidou on Earth
  - ✓ Simple to setup (e.g. no synchronization)
  - ✓ Cost effective (e.g. cost of Moon launches, atomic clocks, …)

Requirements

Mission Objectives

- Mission target: **Moon**
- Navigation strategy: **Relativistic Positioning System**
  - ✓ Only pulse signals (no information transmitted)
  - ✓ No need to synchronize clocks
  - ✓ No relativistic corrections

Simpler than GPS

- Nanosatellites
  - ✓ Low mass
  - ✓ Low volume
  - ✓ Low power requirements

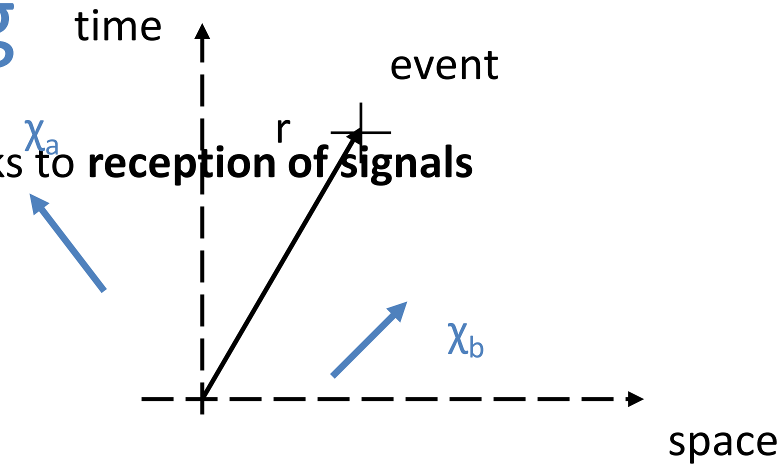
Piggyback launches

Primary Objective

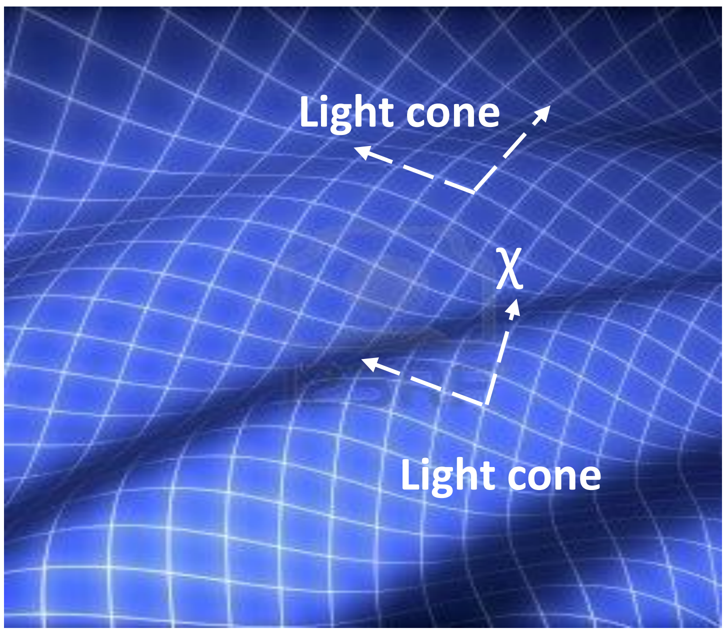
To support human exploration of the Moon by means of an innovative Relativistic Positioning System (RPS) whose accuracy is better than **50 meters**

Relativistic Positioning

- Exploit relativity instead of correcting it!
- 4D grid covering space-time ← drawn thanks to **reception of signals**

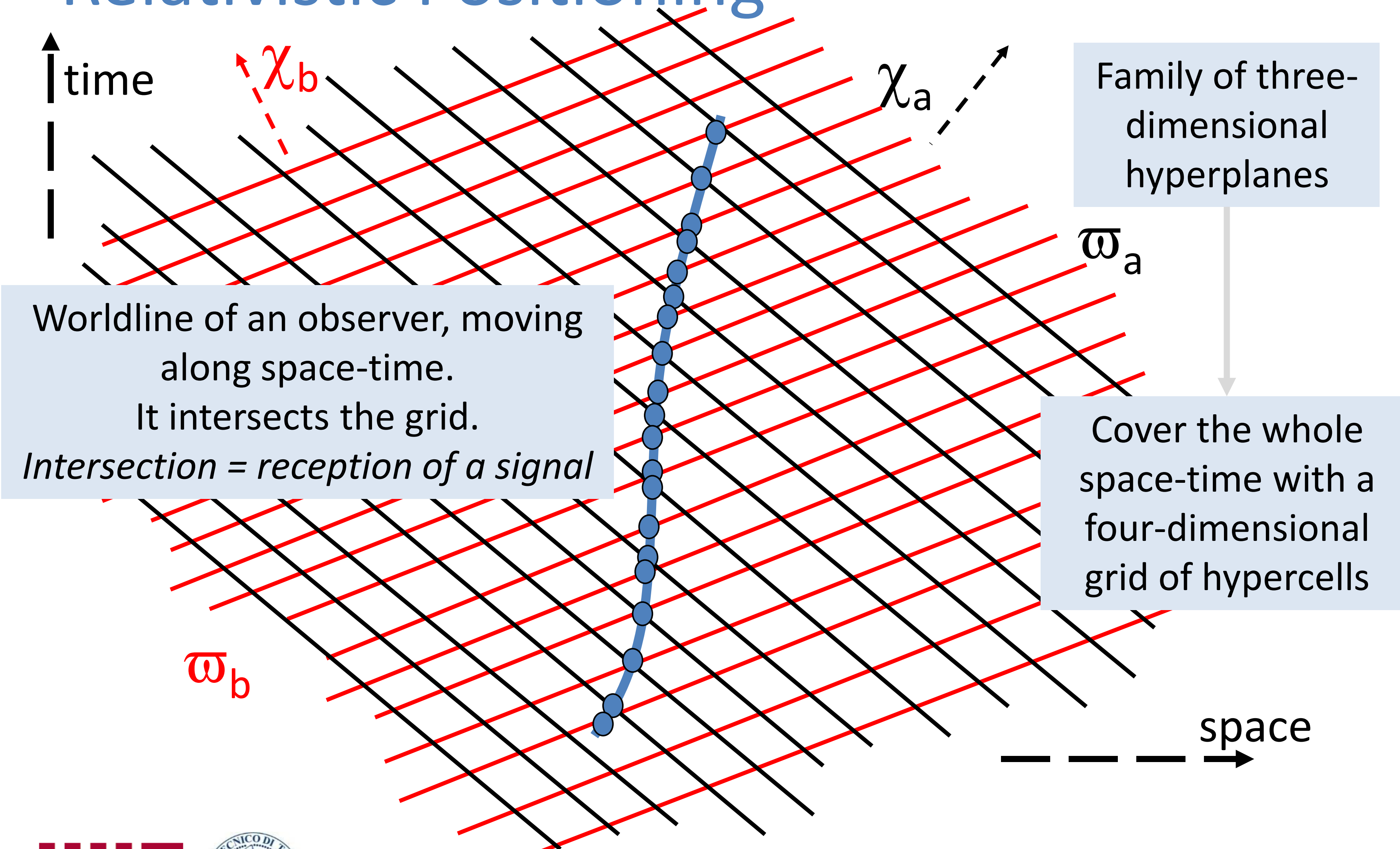


$$r = \frac{t^a}{T_a} c_a + \frac{t^b}{T_b} c_b + \frac{t^c}{T_c} c_c + \frac{t^d}{T_d} c_d$$



- Null geodesics $\boldsymbol{\chi}$
  - ✓ 4D curves along which light and EM signals propagate
  - ✓ Express space-time position of each emitting source
- Vectors χ_a, χ_b, χ_c, χ_d: base of 4D space
- To localize an event **r** in space time
  - ✓ At least 4 sources emitting pulses
  - ✓ Count pulses
  - ✓ Know where sources are and their pulse period

Relativistic Positioning



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Space Segment

Ground Segment

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Conclusions

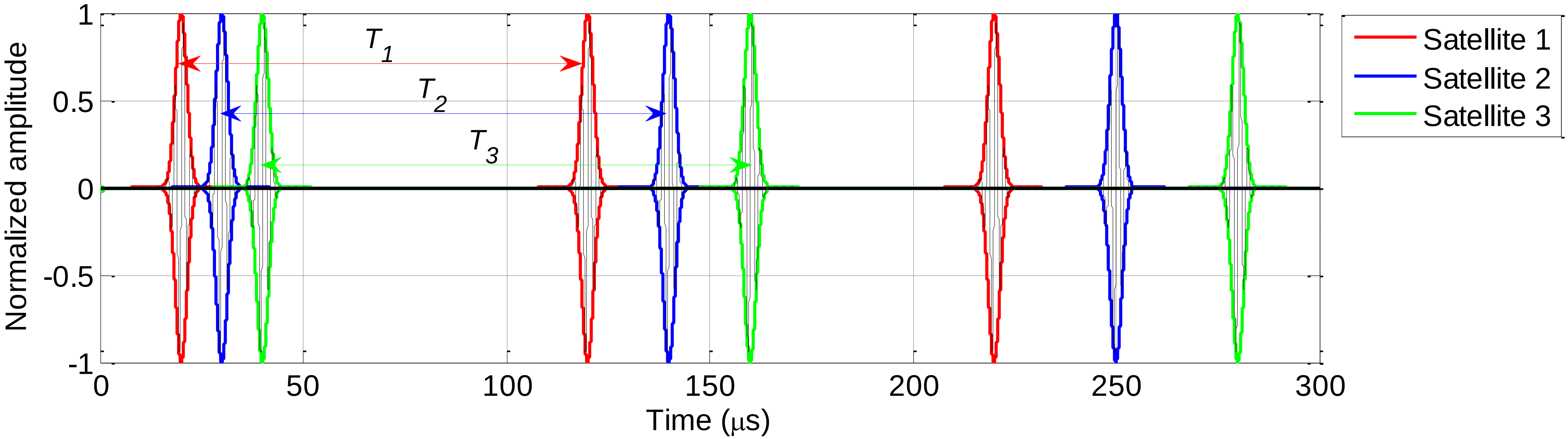
Signals

Orbits

Subsystems

Signals

- Nanosatellites emit periodic *radar-like* electromagnetic Radio Frequency signals
- Gaussian pulse, modulated by a sinusoidal carrier in the S band (2.2 GHz)
- Pulse width: 10 µs
- Each satellite has a different pulse period
- Pulse periods spaced by 10 µs
  - ✓ e.g. T_1=100 µs, T_2=110 µs, T_3=120 µs



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Conclusions

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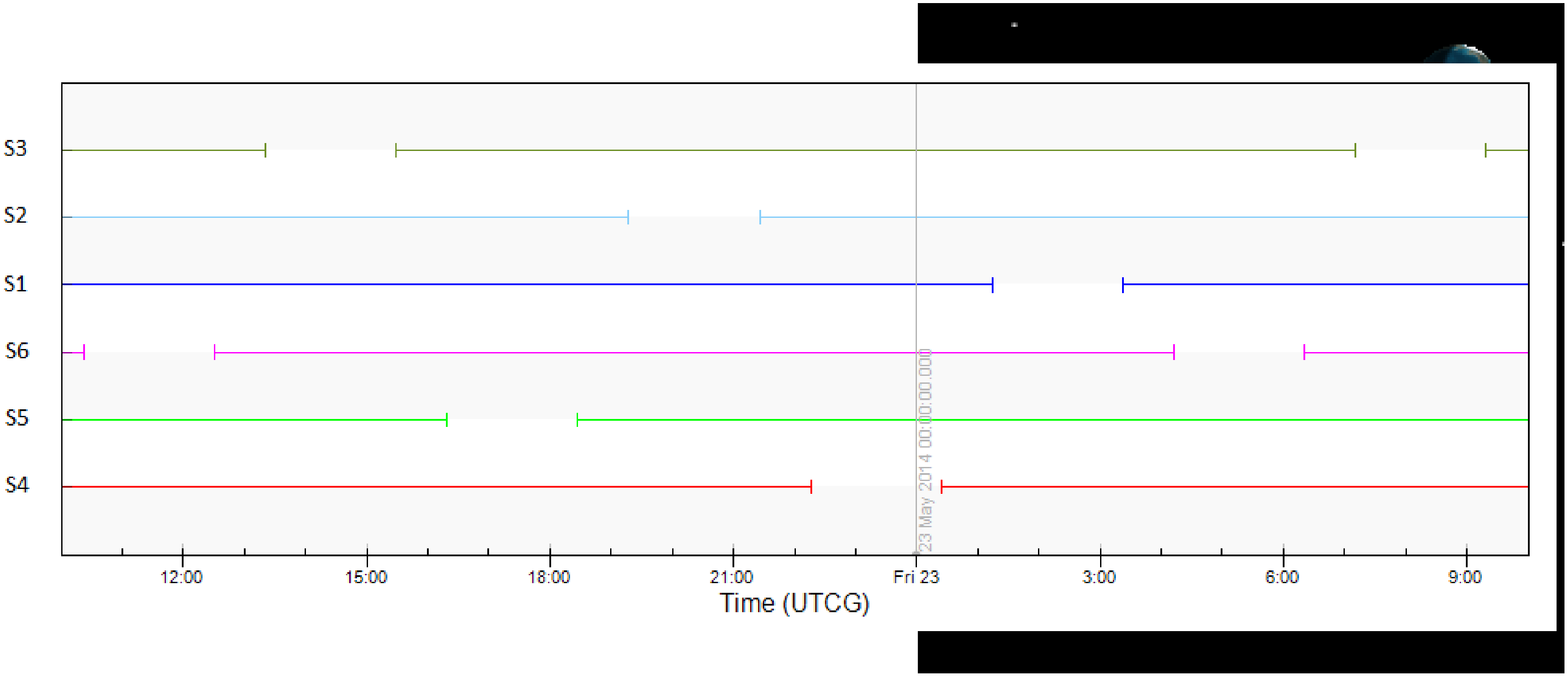
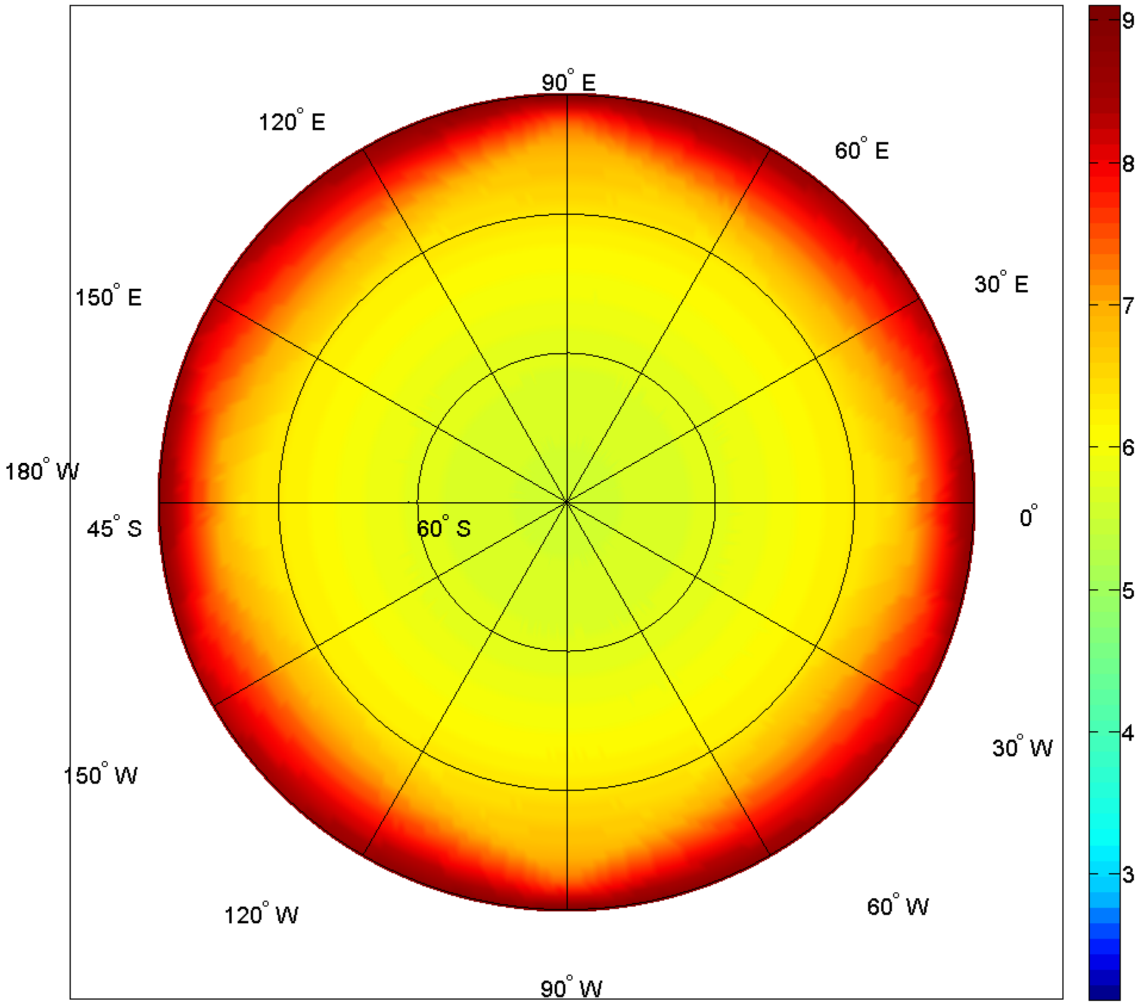
Orbits and Constellation

1. Area of interest: South Pole and surroundings
2. Stable orbits
3. At least 4 satellites in view

Requirements

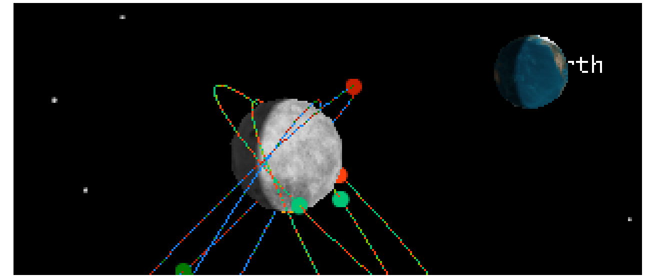


Geometric Dilution Of Precision

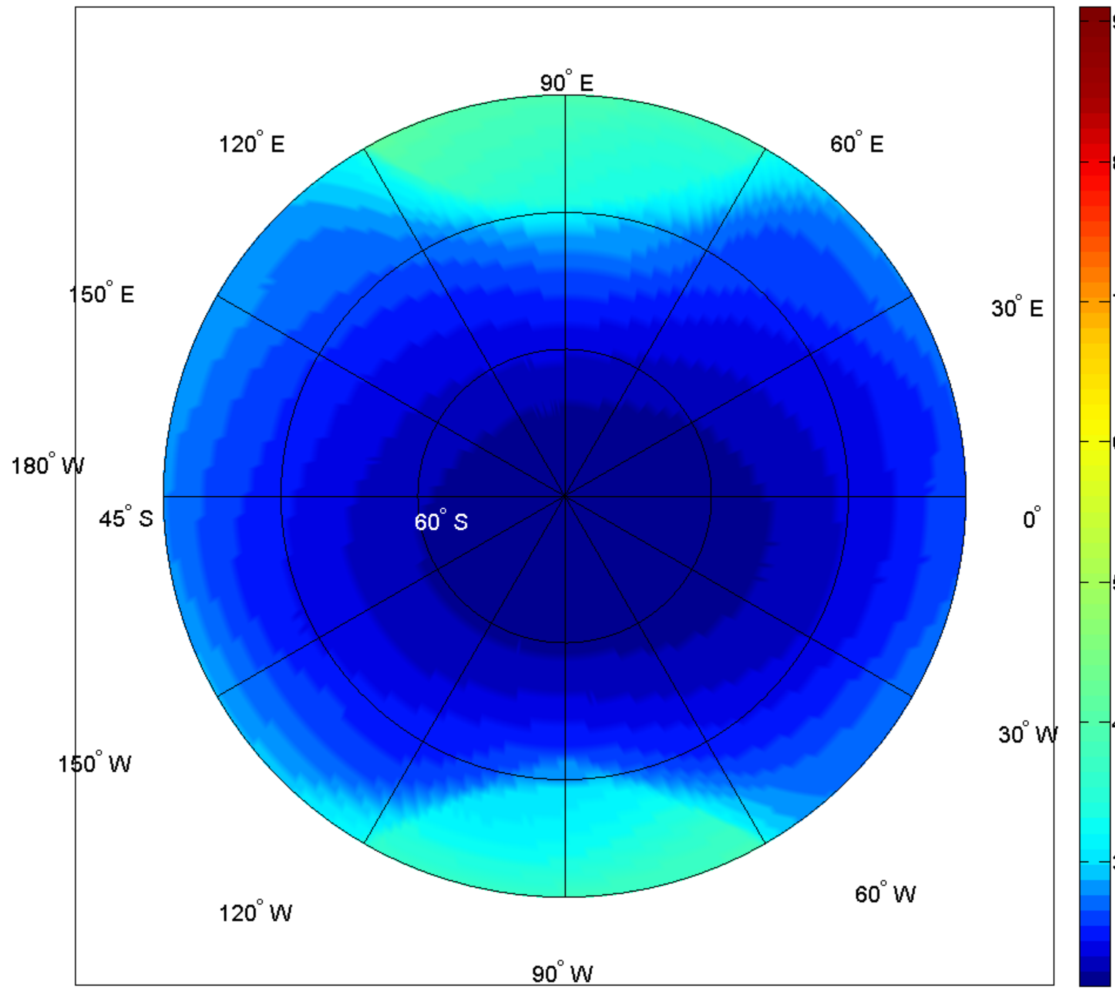


- Accuracy of positioning depends also on a geometry factor
- GDOP is a measure of the composite effect of the relative satellite/user geometry
- Strictly depends on azimuth and elevation of satellites in view
- Expected average GDOP on different points of the lunar surface in a region centered at the South Pole along 18 hours with the 6 satellites constellation

Orbits and Constellation



- Increase accuracy
  - ✓ Geometry
  - ✓ # satellites increase
- Add 6 satellites
  - ✓ a = 5000 km
  - ✓ e = 0.41
  - ✓ i = 45° and 135°
  - ✓ T ≈ 9 hours
  - ✓ M accordingly spaced
- GDOP with **12** satellites



Introduction
Concept of Operations
Space Segment
Ground Segment
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Conclusions

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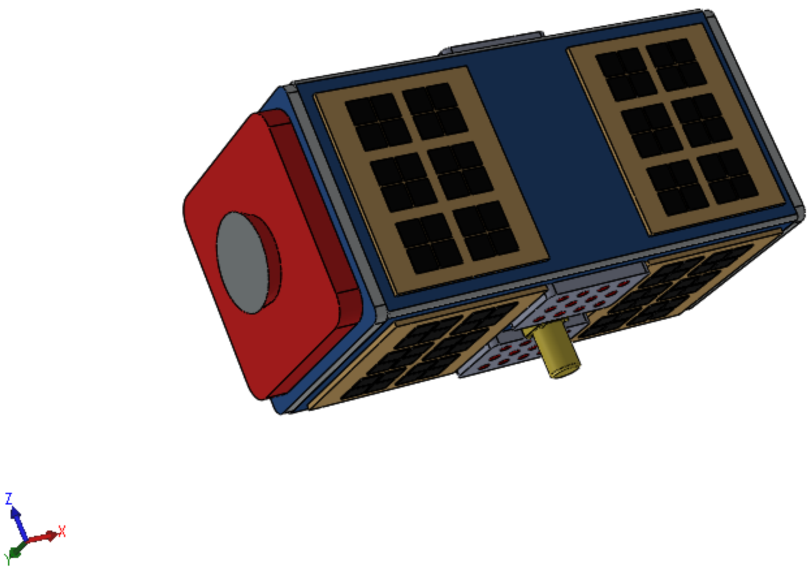
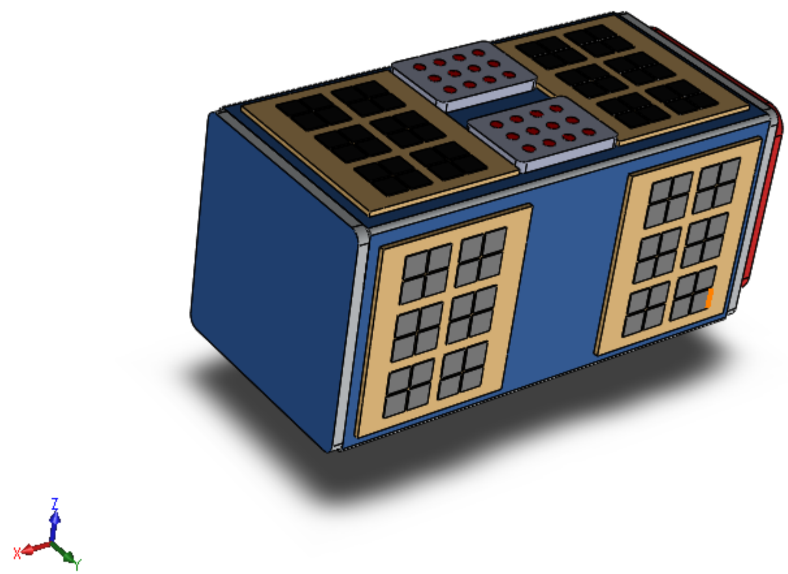
Nanosatellites

- 2U
  - ✓ External size≈10x10x20 cm
  - ✓ Internal volume≈1900 cm^3
- Design maximizes use of COTS

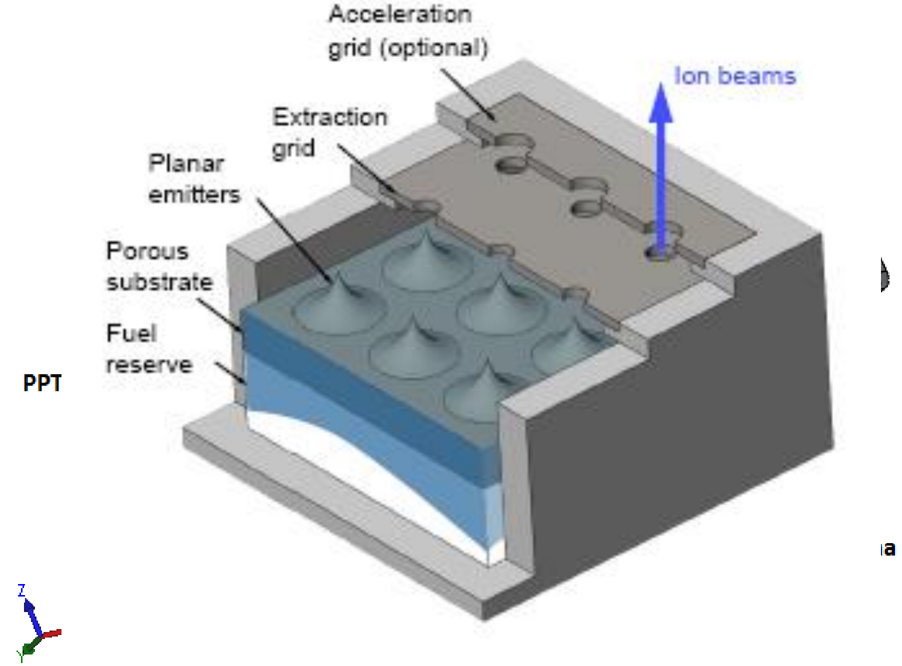
Subsystem	Mass (kg)	Volume (cm^3)	Power (W)
AODCS	1.25	656	1.5
EPS	1.27	397	0.8
OBC	0.10	107	0.4
Clock	0.05	17	0.2
Structure	0.39	-	-
Comm	0.20	74	1.0
Total	**3.90**	**1501**	**4.6**

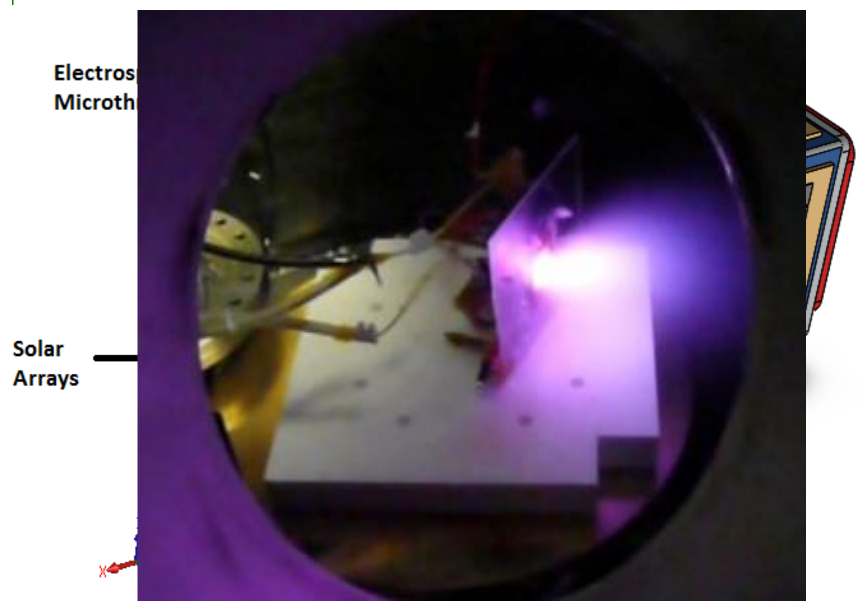




AODCS

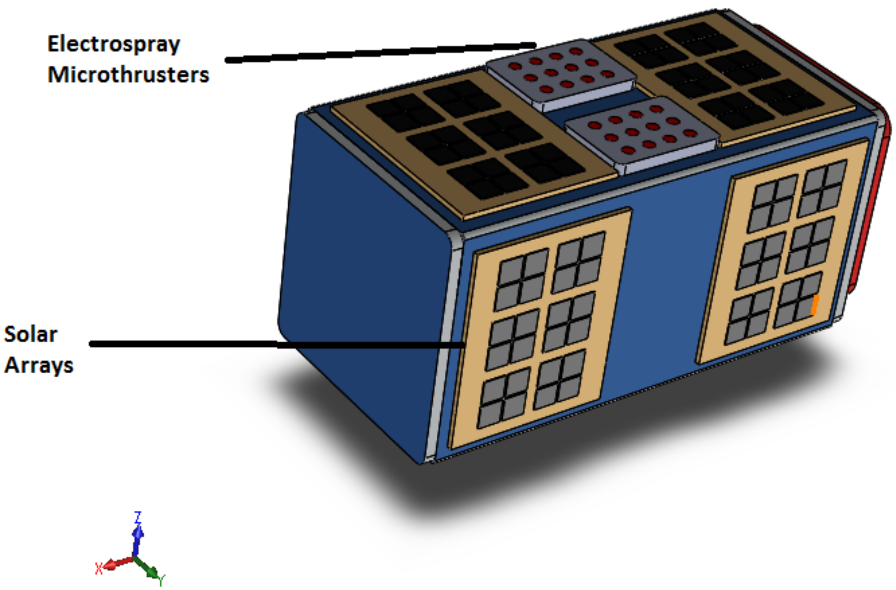
- Sensors
  - ✓ Sun sensors
  - ✓ MEMS gyros (3-axes)
  - ✓ IMU close to center of mass
    - 3-axes accelerometer
    - 3-axes gyro
- Actuators
  - ✓ 1 Pulse Plasma Thruster (PPT)
    - I_{sp}=500-600 s
    - Orbit insertion
    - Boost orbit lifetime (10 years)
    - m_p=0.3 kg
  - ✓ 2 electrospray thrusters
    - Microfabricated Electrospray Arrays
    - I_{sp}=2500-5000 s
    - Fine pointing mode
    - Mission mode
    - m_p=0.1 kg





SPACE SEGMENT

EPS

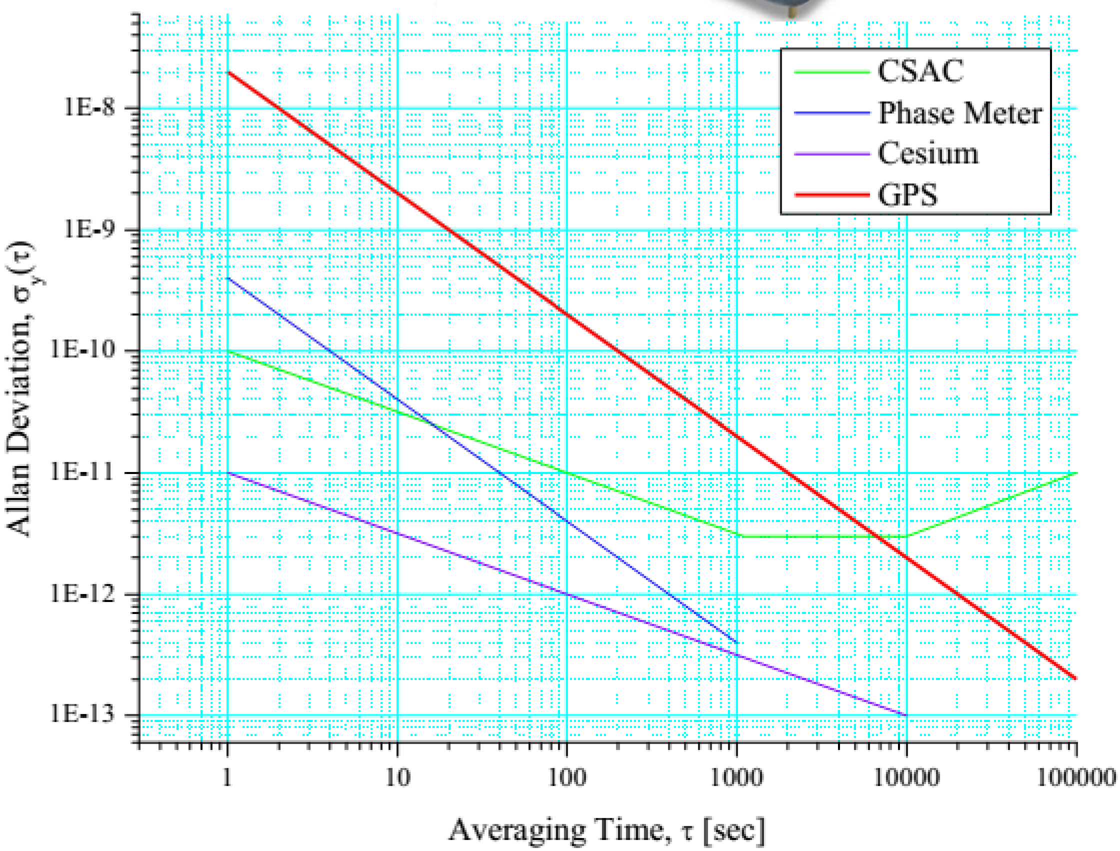


- Requirements
  - ✓ Average power ≈ 5 W
  - ✓ Eclipse: max 1 hour
  - ✓ Daytime: worst case 8 hours
  - ✓ Long eclipses not considered
    - 4-6 hours
    - Only twice a year
    - Standby mode
- Solar panels
  - ✓ 8 GaAs arrays
    - 2.3 W power each
    - 60 g each
- Storage
  - ✓ 2 Li-Ion batteries
    - 40 Wh each
    - 240 g each

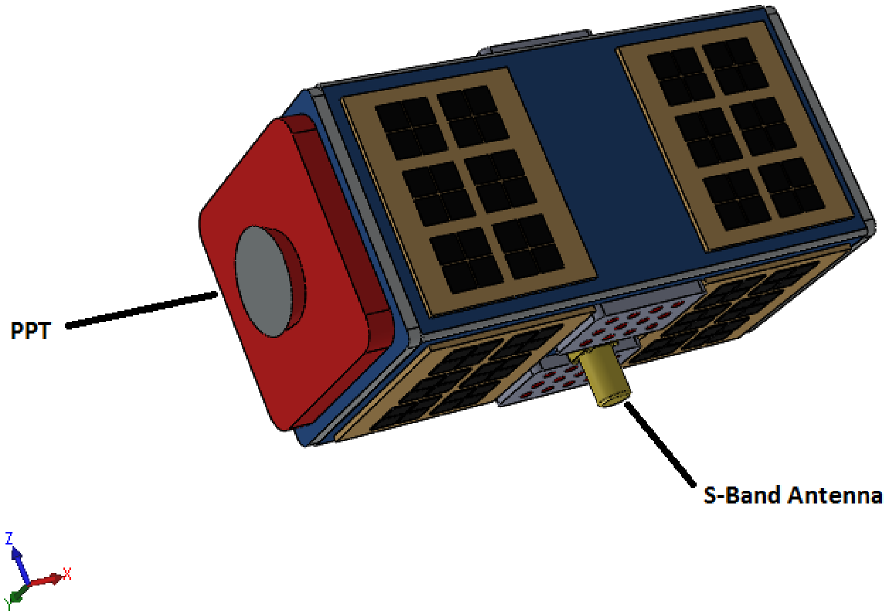
SPACE SEGMENT

Clock

- Scale atomic clock
  - ✓ Quantum™ SA.45s CSAC
  - ✓ 120 mW
  - ✓ 35 g
- Stability
  - ✓ Allan deviation $8x10^{-12}$ at observation time t=1000-10000 s
  - ✓ Positioning error
    - ±9 m
    - Clock update every 3 h
  - ✓ Temperature stability
    - box with a high reflectivity index



Communication



- S-Band: 2.2 GHz
- Antenna
  - ✓ 6 dBi gain
  - ✓ 60° beamwidth
- Power amplifier (transmitter)
  - ✓ 1 W
- Link
  - ✓ False alarm probability of 10^{-5}
  - ✓ Detection probability of 0.9

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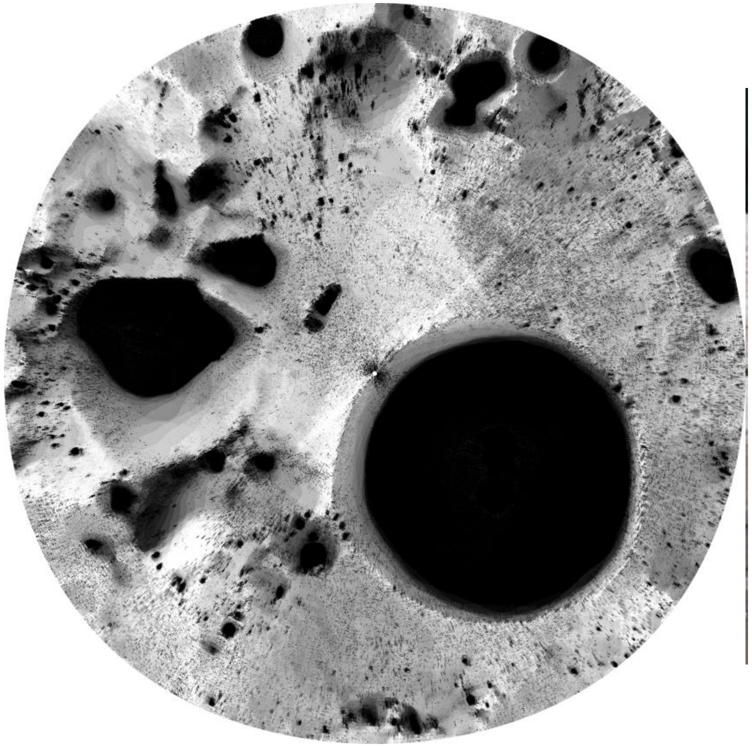
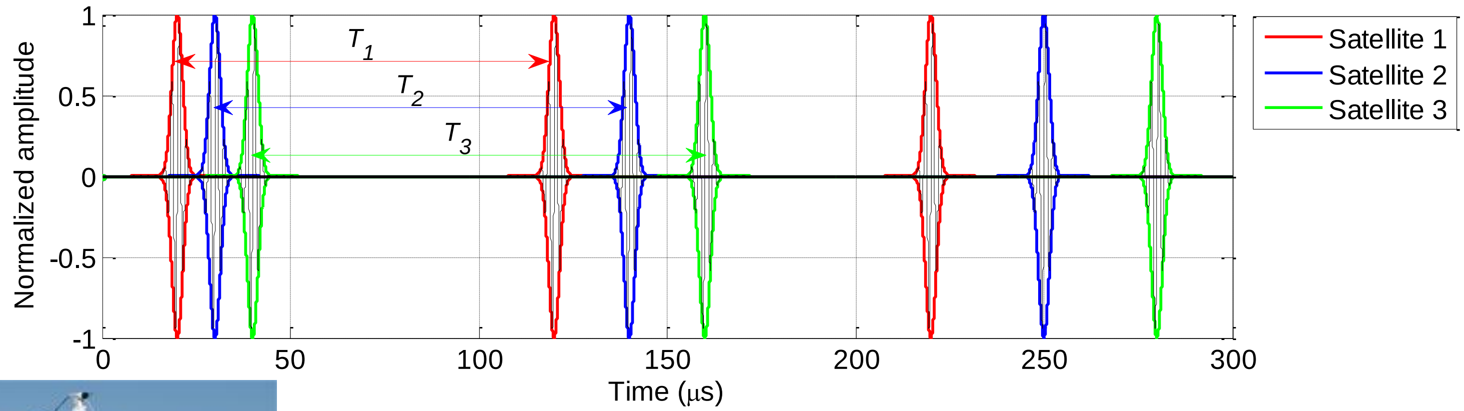
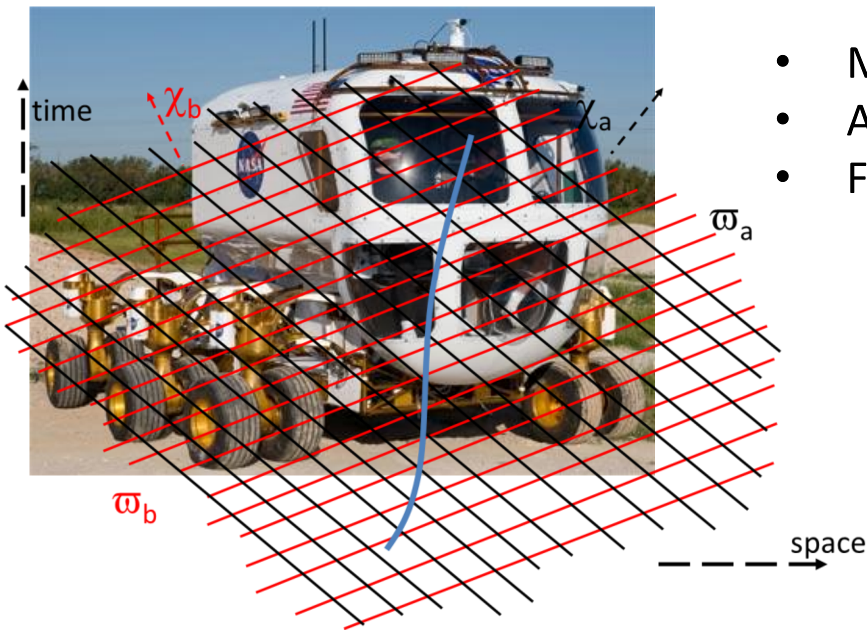
Ground Station

- Lunar base at South Pole
- High-performance antennas to track the satellites in space-time
  - ✓ Ephemerides
  - ✓ Signal periods
- Updates to be transmitted to users
  - ✓ Every 3 hours
  - ✓ Data relay satellites orbiting around the Moon
- Operations
  - ✓ Astronauts at lunar base (especially EVA support)
  - ✓ Team of 3-4 people on Earth



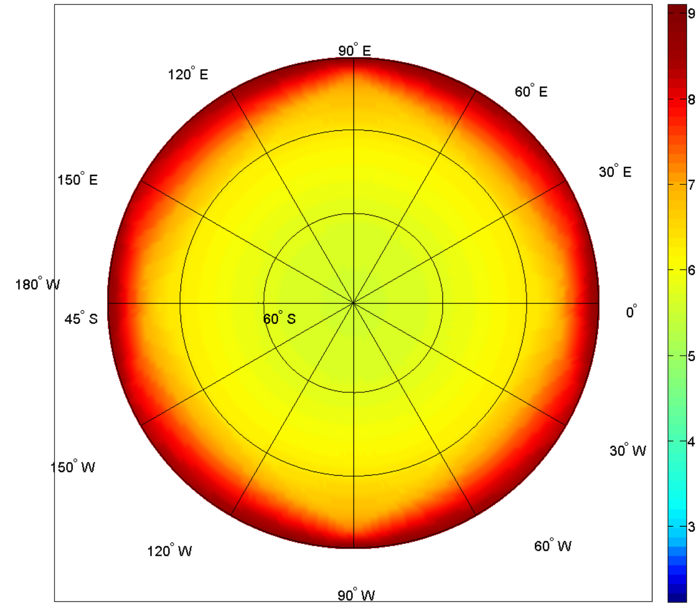
Users

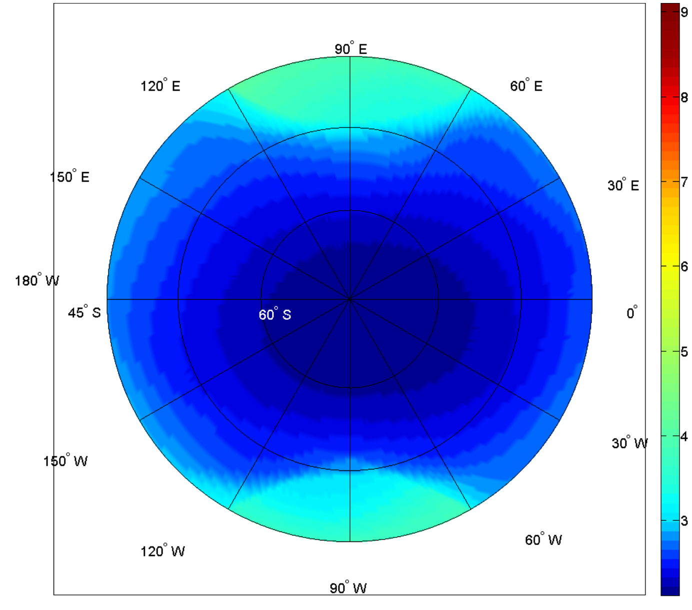




- Manned/unmanned rovers
- Antenna: gain 30 dBi and system temperature 290 K
- Front-end + Analog to Digital Converter + **Clock**
  - ✓ Detect pulses in time and frequency domain
  - ✓ Identify the emitting satellite by computing the pulses period
  - ✓ Compute the time interval between two signals from two different emitters

Positioning Accuracy





- Root-sum-square of different contributions
  - ✓ Satellite clock phase error (±9 m)
  - ✓ Ephemeris error (radial ±1 m, along track ±4 m, cross track ±2 m)
  - ✓ Receiver noise and resolution (±3 m)
  - ✓ Multipath reflections (±1 m)
- Multiplied by GDOP
  - ✓ Error **< 100 m** in 6 satellite constellation
  - ✓ Error **< 50 m** in 12 satellite constellation

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Concept of Operations
Space Segment
Ground Segment
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Project Schedule

Year	2014				2025				2026				2027				2028				2029				2031				2039				2041			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
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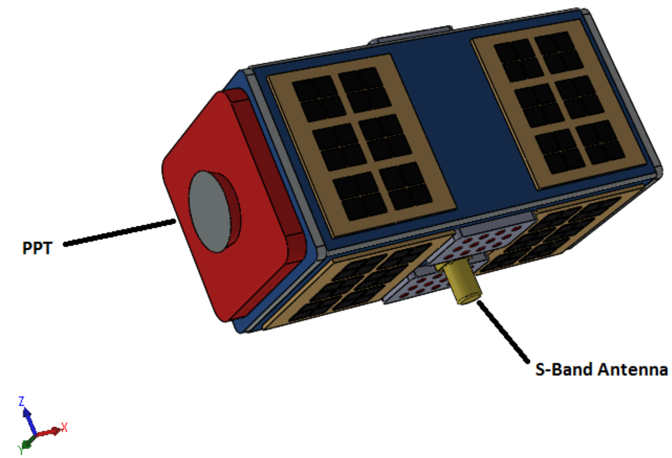
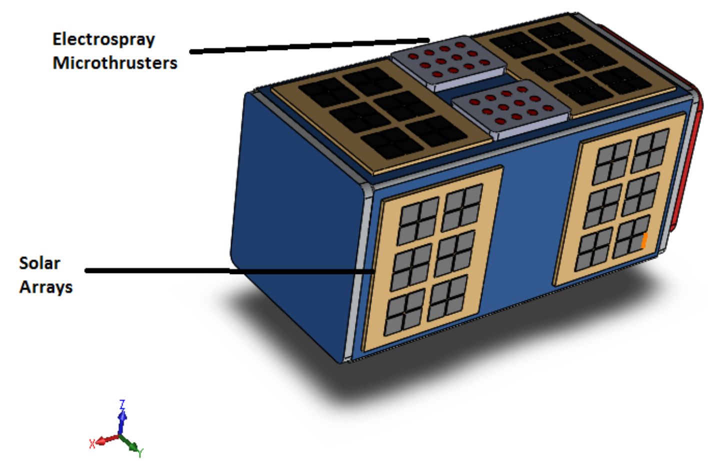
Cost Analysis

- Cost of one nanosatellite ≈ 150 k$
- Design, assembly, development, integration and testing
  - ✓ Team of 15-20 people
  - ✓ 4-5 years
  - ✓ Average salary of 70-100 k$ per year per person
  - ✓ Prototype cost ≈ 0.5-1 M$
- Launch costs for 12 cubesats ≈ 0.6 M$
- Operations
  - ✓ Team of 3-4 people
  - ✓ 10 years
- Disposal: de-orbit propellant already considered
- Total ≈ **15 M$**





Introduction
Concept of Operations
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Ground Segment
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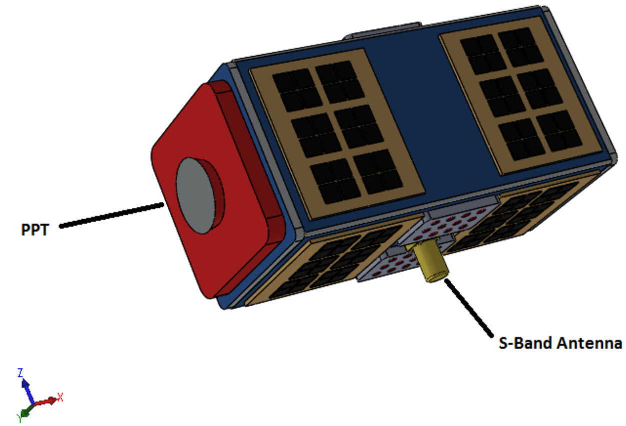
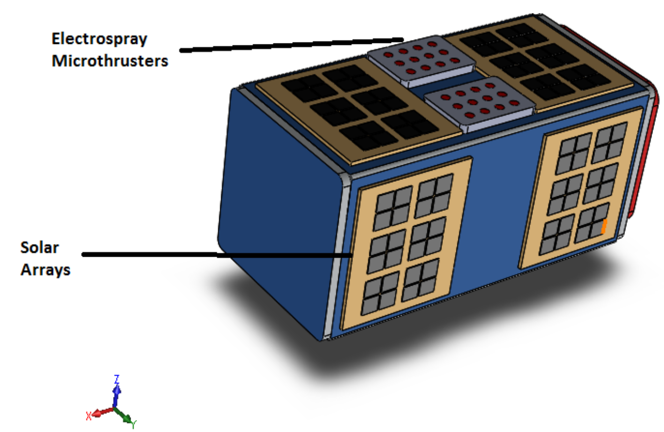
Potential Risks and Solutions

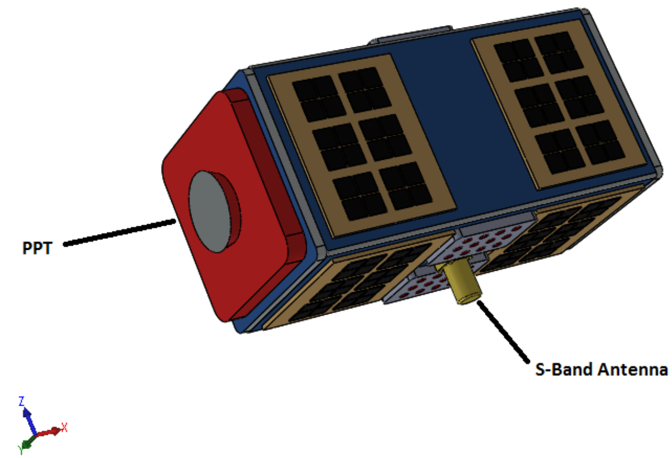
- Competition with existing technologies
  - ✓ Earth GPS on the Moon
    - Good results
    - Limited availability
  - ✓ LRPS
    - Reduced complexity and low cost
    - Very good coverage and GDOP
    - Also on other planets
- False alarm and detection probabilities
  - ✓ High gain receiver antennas
  - ✓ Advanced signal processing techniques
- Orbit stability
  - ✓ Mascons not well determined
  - ✓ Orbit control system included
- Maintenance
  - ✓ Redundancies in constellation size

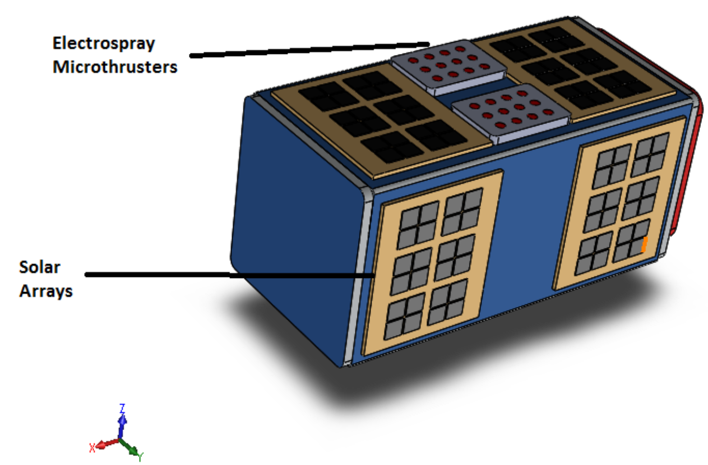




Project Sustainability

- Components Off The Shelf
  - ✓ Low development and construction costs
  - ✓ Standardized size
- Launch
  - ✓ Piggyback
  - ✓ Decrease in launch costs
- Not particular ground infrastructure to build
- Mission architecture easily extendable to cover the whole Moon
- Support spacecraft landing on the Moon
- A similar idea could be used to improve GPS coverage at Earth poles





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Thank you!
ありがとうございました

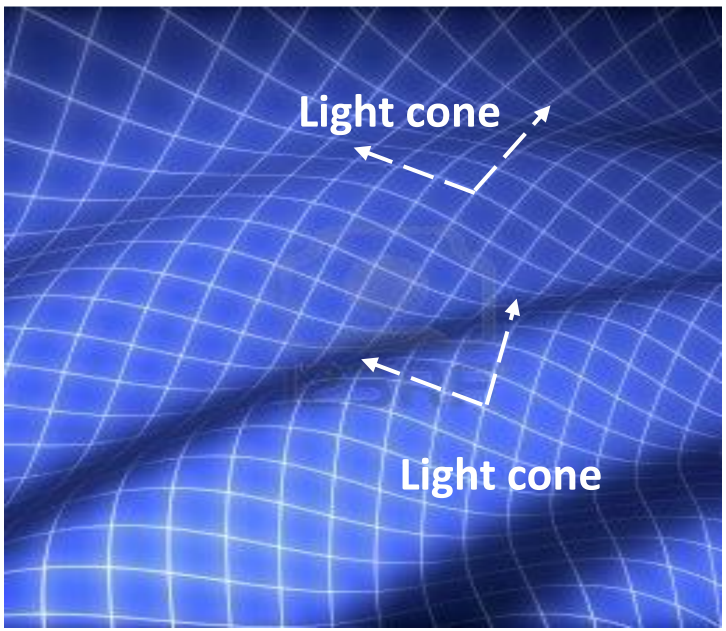
Back up slides

Relativistic Positioning

Relativistic Positioning

- Exploit relativity instead to correct it!
- Four dimensional grid covering space-time ← drawn thanks to reception of signals
- No need to synchronize clocks
- No need to define origin of time
- No relativistic corrections

Simpler than GPS



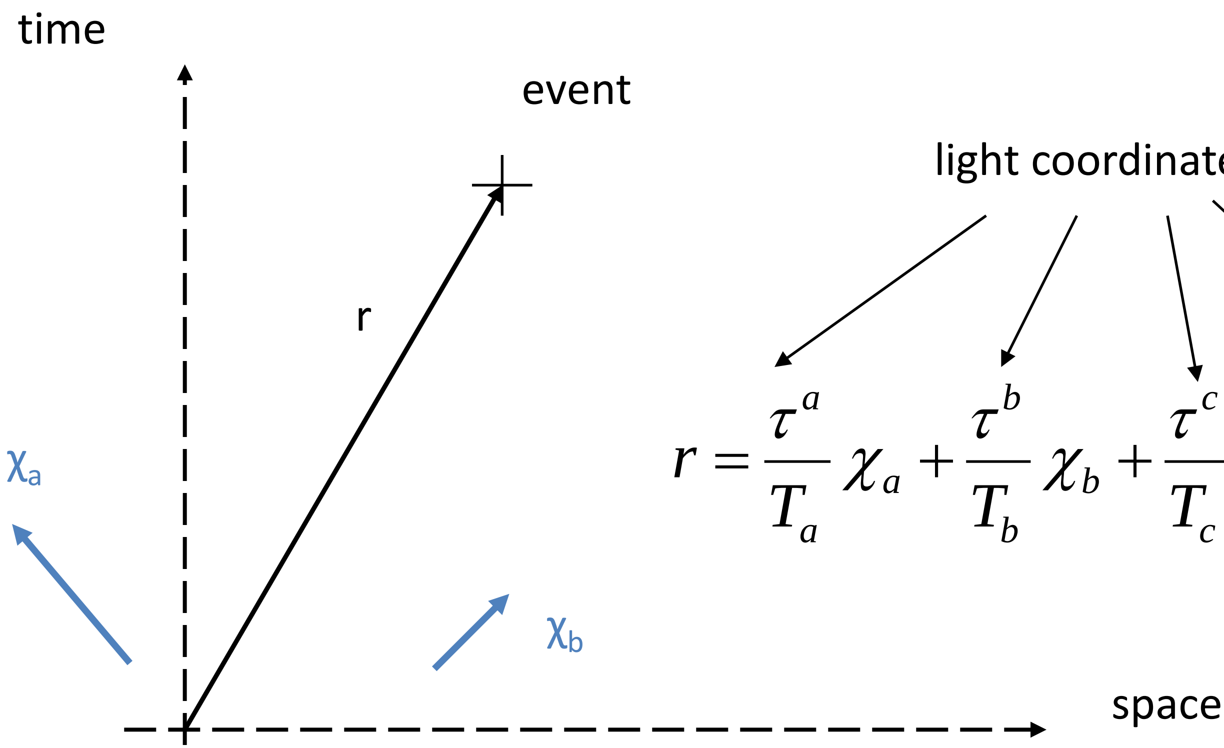
- Null geodesics
  - ✓ Null tangent (wave) vector

$$\underbrace{\chi = cT}_{\text{Period of the signal}}\underbrace{\left(1, \cos\alpha, \cos\beta, \cos\gamma\right)}_{\text{Direction cosines}} = cT\left(1, \vec{n}\right)$$

$$\chi \cdot \chi = 0$$

- Vectors χ_a, χ_b, χ_c, χ_d: base of 4D space

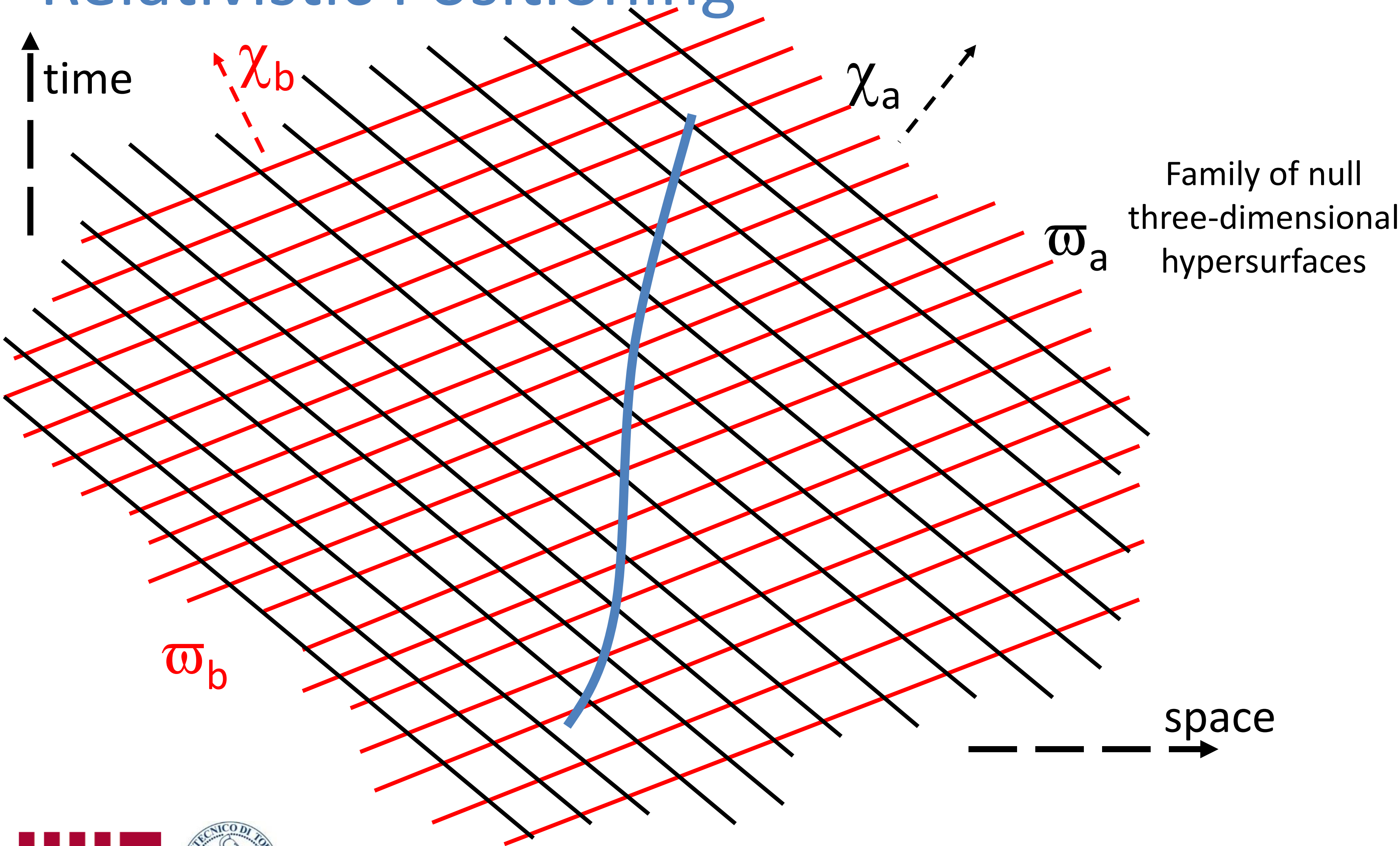
Relativistic Positioning



light coordinates

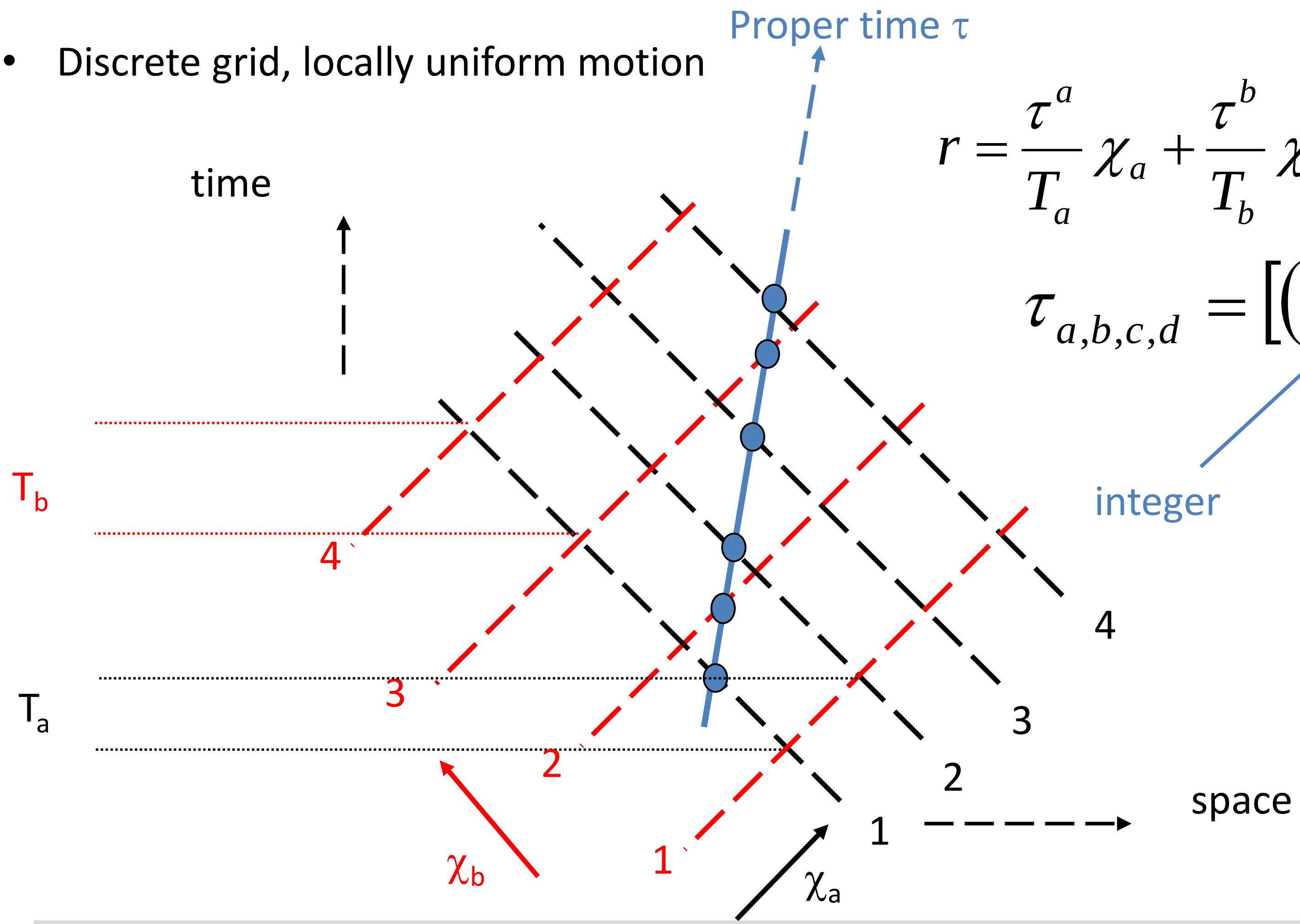
$$r = \frac{\tau^a}{T_a}\chi_a + \frac{\tau^b}{T_b}\chi_b + \frac{\tau^c}{T_c}\chi_c + \frac{\tau^d}{T_d}\chi_d$$

Relativistic Positioning



Relativistic Positioning

- Discrete grid, locally uniform motion



$$r = \frac{\tau^a}{T_a}\chi_a + \frac{\tau^b}{T_b}\chi_b + \frac{\tau^c}{T_c}\chi_c + \frac{\tau^d}{T_d}\chi_d$$

$$\tau_{a,b,c,d} = \left[(n + x)T\right]_{a,b,c,d}$$

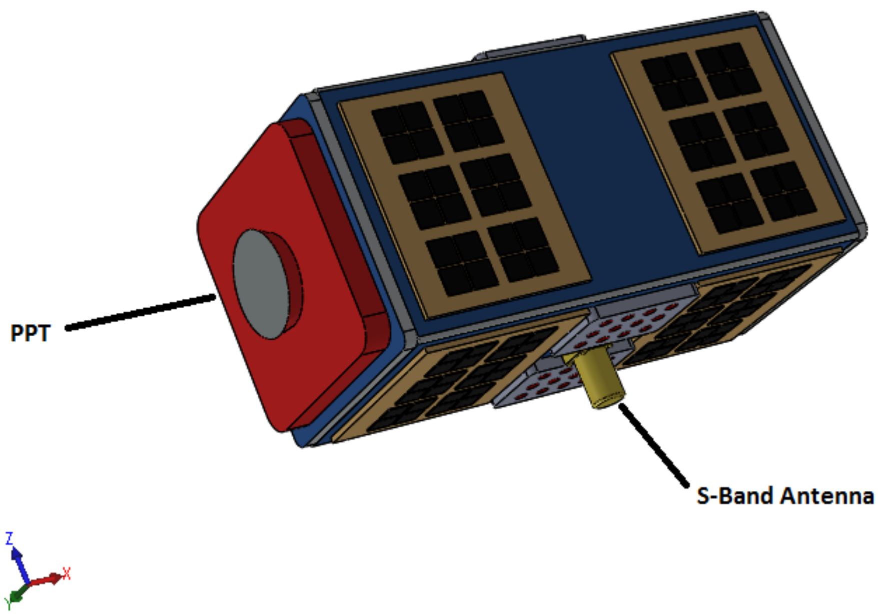
integer

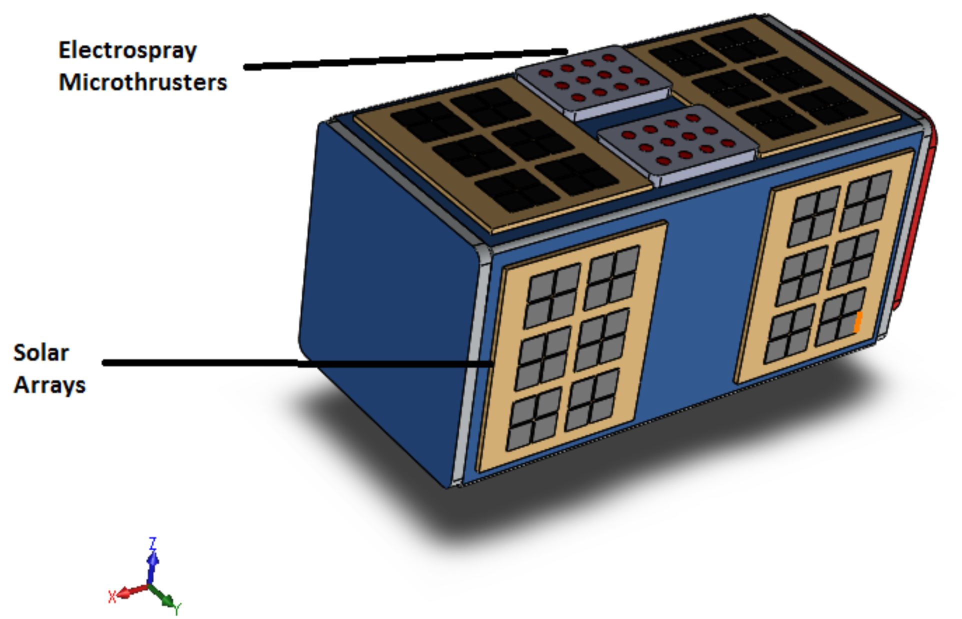
From simple linear equations

AODCS

AODCS

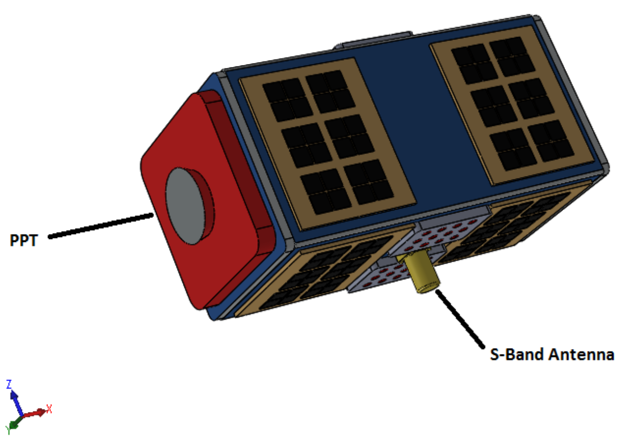
- Attitude and orbit perturbations
  - ✓ Third body effect due to Earth and Sun gravity
  - ✓ Forces due to solar radiation and wind
  - ✓ Commanded thrust forces

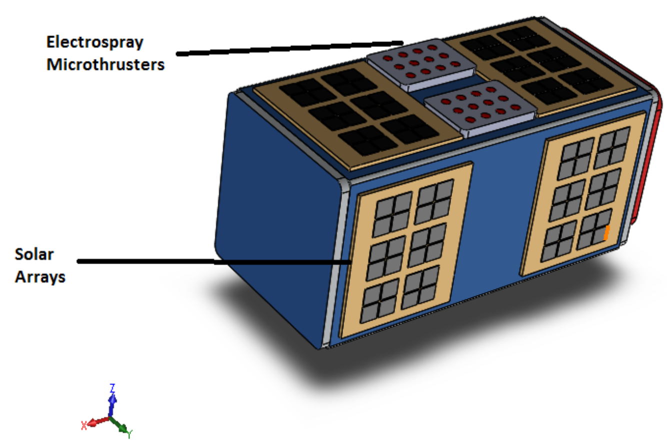




AODCS

- Attitude stabilization maneuvers
  - ✓ 10 m/s per maneuver
  - ✓ 3 times a year: 2 after long eclipses + margin
- Station-keeping in lunar frozen orbit
  - ✓ 40 m/s/year
- Fine attitude control in LRPS mission phase
  - ✓ 10 m/s/year
- Nanosatellite disposal
  - ✓ 10 m/s





The 3rd
Mission Idea Contest
for Micro/Nano-satellite Utilization

AODCS: control modes

- **Separation and de-tumbling mode**.
  - ✓ attitude uncertainty: about 5°
  - ✓ spinning velocity about the x-axis
  - ✓ accurate antenna orientation is not required
  - ✓ colloidal microthrusters reduce the tumbling motion under a certain threshold.
- **Attitude and orbit acquisition mode.**
  - ✓ attitude and orbit starting points uncertainty: 5°, at most.
- **Orbit insertion mode.**
  - ✓ error affecting position and velocity vectors below a defined threshold.
- **Fine attitude acquisition and pointing mode.**
  - ✓ Nadir-pointing control strategy
  - ✓ attitude uncertainty: 1-2°
- **LRPS Mission mode.**
  - ✓ on-board systems checked and working.
  - ✓ orbit control system (firing the PPT)
- **Safe mode.**
  - ✓ eclipse periods/in case of failure
  - ✓ minimum amount of power

AODCS: proposed hardware – SENSORS 1/2

- ✓ Sun sensors
  - Photo-diodes integrated in the solar panels
  - Current: 170 uA
  - Field of view: 114°
  - Update Rate: >10 Hz (limited by ADC)
  - Accuracy: <0.5°
- ✓ MEMS gyros integrated in the solar panels (3-axes)
  - Range: 80 °/s
  - Sensitivity: 0.00458 °/s
  - Bias stability: 0.016 °/s
  - Vcc: 5 V
  - Current: 44 mA

AODCS: proposed hardware – SENSORS 2/2

- ✓ IMU (6 Degrees of Freedom on a single, flat board):
  - ITG-3200 - triple-axis digital-output gyroscope
    - Digital-output X-, Y-, and Z-Axis angular rate sensors (gyros) on one integrated circuit
    - Digitally-programmable low-pass filter
    - Low 6.5mA operating current consumption for long battery life
    - Wide VDD supply voltage range of 2.1V to 3.6V
    - Standby current: 5μA
    - Digital-output temperature sensor
    - Fast Mode I2C (400kHz) serial interface
    - Optional external clock inputs of 32.768kHz or 19.2MHz to synchronize with system clock
  - ADXL345 - 13-bit resolution, ±16g, triple-axis accelerometer
    - 1.8V to 3.6V supply
    - Low Power: 25 to 130uA @ 2.5V
    - SPI and I2C interfaces
    - Up to 13bit resolution at +/-16g
    - Activity/Inactivity monitoring
    - Free-Fall detection
  - Outputs of all sensors processed by on-board ATmega328 processor and sent out via a serial stream
  - 3.5-16VDC input
  - Dimensions: 1.1" x 1.6" (28 x 41mm)

AODCS: attitude perturbations (torques) 1/2

✓ Non-spherical gravitational field (gravity-gradient torques)

$$\boldsymbol{M_g(t)} = \frac{3}{2}\boldsymbol{\omega_0}^2 \begin{bmatrix} (\boldsymbol{J_3} - \boldsymbol{J_2}) \sin 2\varphi \cos^2 \boldsymbol{\vartheta} \\ -(\boldsymbol{J_1} - \boldsymbol{J_3}) \sin 2\varphi \cos \boldsymbol{\vartheta} \\ -(\boldsymbol{J_2} - \boldsymbol{J_1}) \sin 2\varphi \sin \boldsymbol{\vartheta} \end{bmatrix}, \ \boldsymbol{\omega_0}^2 = \frac{\boldsymbol{\mu_M}}{\boldsymbol{a}^3}$$

Where:

- $M_g(t)$: perturbing torque affecting the s/c
- J_1, J_2, J_3 : inertia of the s/c about the principal axes (hyp. Body axes are principal)
- $\varphi, \boldsymbol{\vartheta}$: s/c attitude angles
- $\boldsymbol{\mu_M}$: Lunar gravitational constant
- a: orbit semi-major axis

✓ External particles (solar wind and Lunar residual atmosphere)

Obs: Disturbance aerodynamic torques generated iif CoM ≠ CoP

$$\vec{F}_D = \sum_{k=1}^{n} \vec{F}_{Dk} = -\frac{1}{2}\rho |\vec{v}_r|^2 \sum_{k=1}^{n} C_{Dk} \max\left(\cos\alpha_k, 0\right) A_k \vec{e}_v$$

$$\cos\alpha_k = \vec{n}_k \cdot \vec{e}_v$$

$$\vec{M}_D = \sum_{k=1}^{n} \vec{a}_k \times \vec{F}_{Dk}$$

Where:

- M_D: perturbing torque affecting the s/c
- F_D: perturbing force affecting the s/c
- ρ: atmosphere density
- v_r: relative velocity
- n: number of surfaces of the s/c
- A_k: area of the k-th surface of the s/c
- $\overrightarrow{e_v}$: incident particle direction
- C_{Dk} : drag coefficients

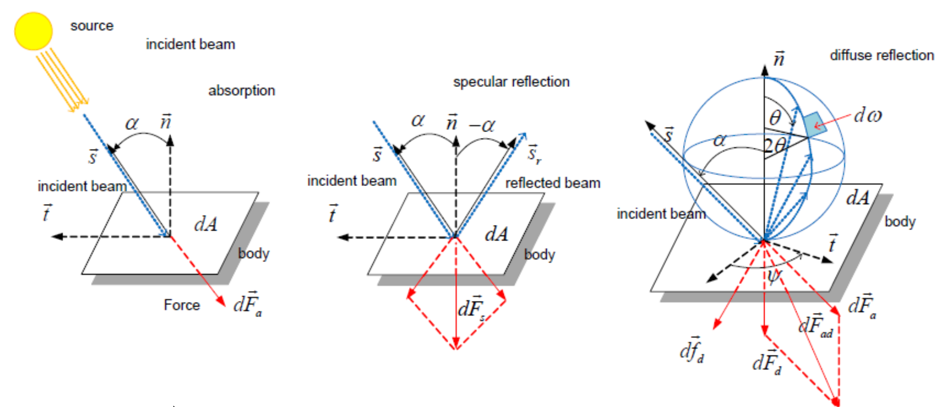
AODCS: attitude perturbations (torques) 2/2

✓ **Electromagnetic waves (photons) from Sun (radiation solar pressure) and Moon emitted/reflected radiation**

Obs: Disturbance torque generated iif CoM ≠ CoP

The resulting pressure depends on:
- the vehicle geometry,
- the radiation direction in the body frame,
- the optical properties of the surface.

The incident electromagnetic may be:
- completely absorbed by the surface,
- specularly reflected,
- diffusely reflected in any direction,
- transmitted.



$$\vec{F}_r = \sum_{k=1}^{n} \vec{F}_{rk} = -p\sum_{k=1}^{n}\Big(\big(1-C_{sk}\big)\vec{s} + 2\big(C_{sk}\max\left(\cos\alpha_k, 0\right) + C_{dk}/3\big)\vec{n}_k\Big)\max\left(\cos\alpha_k, 0\right)A_k$$

$$\cos\alpha_k = \vec{s}\cdot\vec{n}_k$$

$$\vec{M}_r = \sum_{k=1}^{n}\vec{a}_k \times \vec{F}_{rk}$$

Where:
- M_r: perturbing torque affecting the s/c
- F_r: perturbing force affecting the s/c
- n: number of surfaces of the s/c
- A_k: area of the k-th surface of the s/c
- C_{sk}, C_{dk}: specular and diffusion coefficients
- α: angle of incidence
- p: pressure of the electromagnetic radiation
- $\overrightarrow{n_k}$: outer normal direction of the k-th surface of the s/c

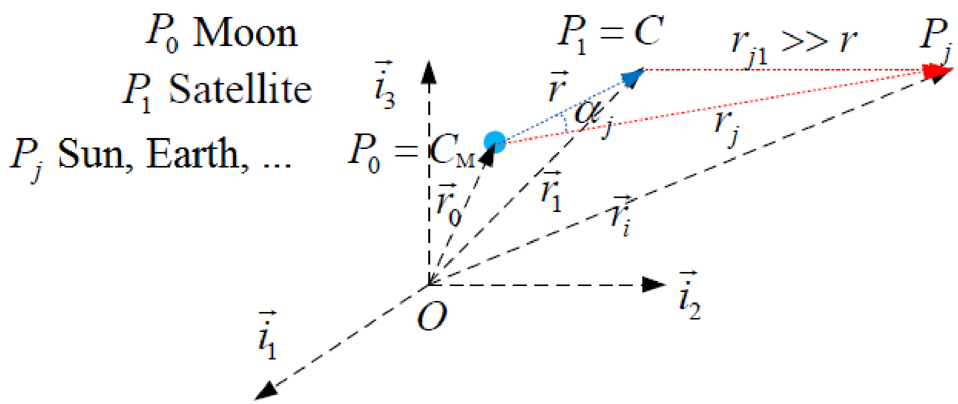
AODCS: orbit perturbations (forces)

✓ **Third-body effects: Earth and Sun**

$$F(t) \cong m \sum_{j=2}^{n-1} \left(-\frac{\mu_j r}{r_j^{\,3}} \left(\frac{\boldsymbol{r}}{r} - 3\frac{\boldsymbol{r_j}}{r_j} \cos\alpha \right) \right)$$

Where:

- F(t): perturbing force affecting the s/c
- m: s/c mass
- n: number of space bodies taken into account
- μ_j: j-th body gravitational constant
- r_j: j-th body radius
- r: satellite radius
- α: angle of view



Geometry of n-body problem.

✓ **Aerodynamic forces and wind (residual atmosphere)**

$$F = -\frac{1}{2}\rho v_r^{\,2} \sum_{k=0}^{n-1} \left(C_{n,k}\overrightarrow{n_k} + C_{v,k}\overrightarrow{e_v} \right) A_k$$

Where:

- F: perturbing force affecting the s/c
- ρ: thermosphere density
- v_r: relative velocity
- n: number of surfaces of the s/c
- A_k: area of the k-th surface of the s/c
- $\overrightarrow{n_k}$: outer normal direction of the k-th surface of the s/c
- $C_{n,k}, C_{v,k}$: mixed aerodynamic coefficients

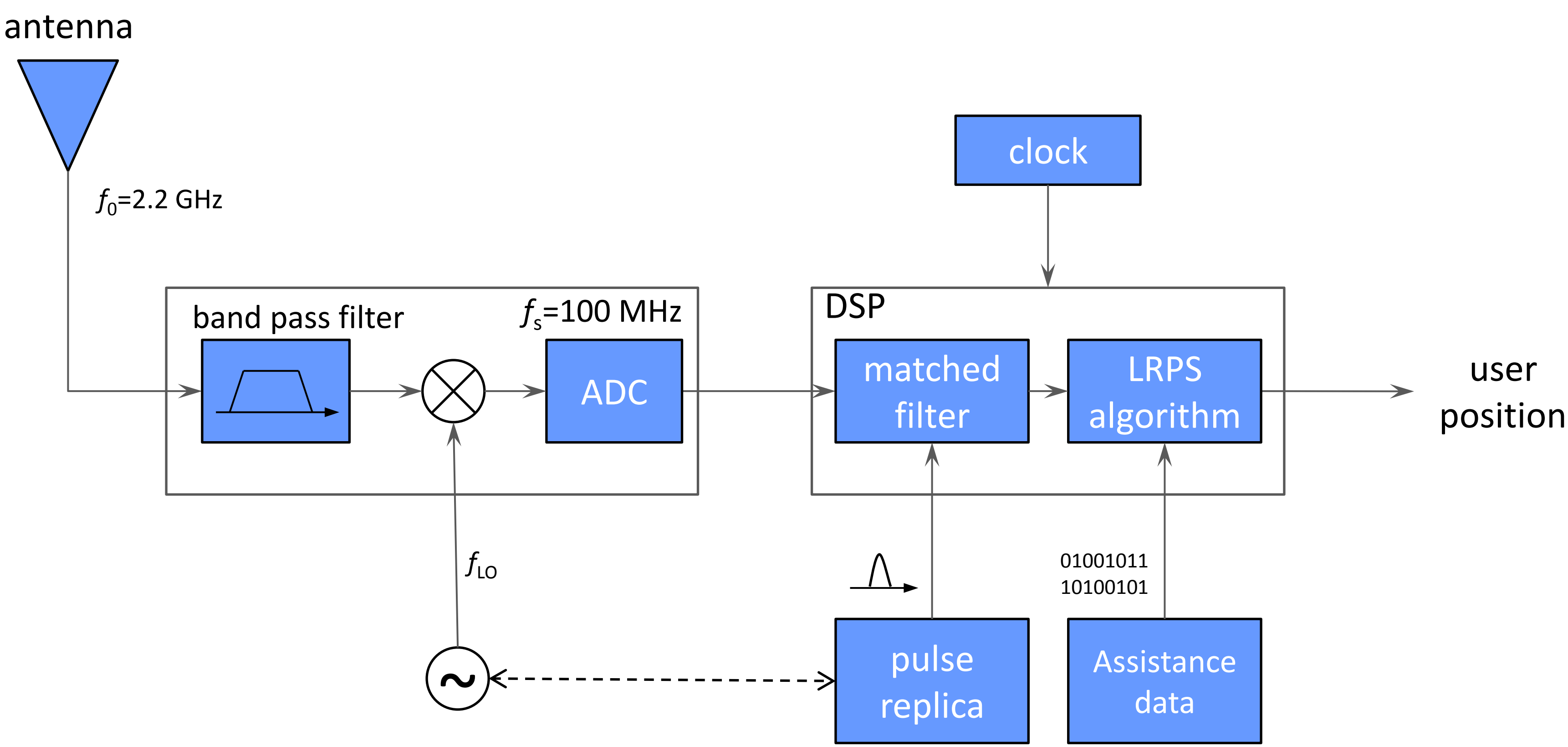
EPS

EPS: solar panels

- ✓ 8 GaAs arrays
- ✓ Single panel properties:
  - Power: 2.3 W
  - Operational temperature: -40 ^{o}C to +85 ^{o}C
  - Dimensions: 82.5mm x 98.0mm x 2.1mm
  - Height of connectors: 4.8 mm
  - Height of gyro: 5.5 mm
  - Panel thickness: 2.15 mm
  - Mass: 60 g
- ✓ Single panel further features:
  - Two series-connected AzurSpace 3G-30 space qualified triple junction solar cells + protection diodes
  - Integrated magnetorquer of 1.6 m^2
  - Coarse sun sensor
  - Temperature sensor
  - Gyro-scope model ADIS16251 (0.004 ^{o}/s)
  - Top/Bottom or side panel version

Signal

Receiver block scheme

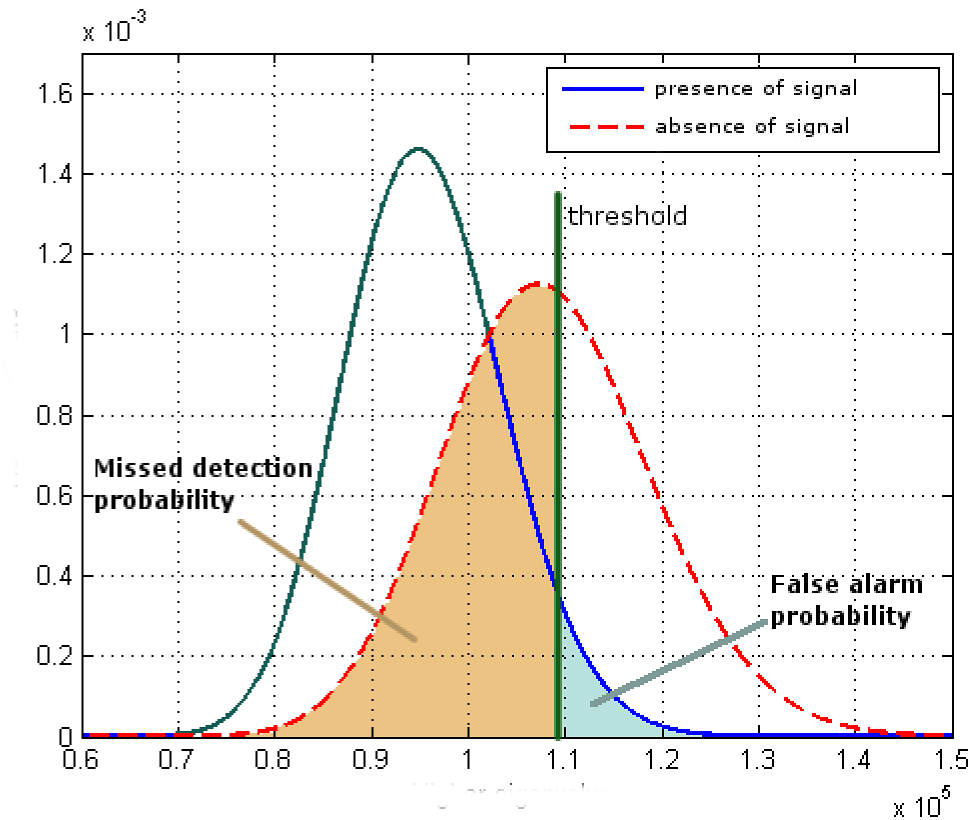


Signal acquisition strategies

Synchronization problem:

1. CDMA modulation (similar to GPS)
   - ✓ Orthogonal codes
   - ✓ Despreading gain
   - ✓ Robust to AWGN
   - ✓ Allows signal identification
   - ✓ More complex receiver
2. Edge detector
   - ✓ Time domain analysis
   - ✓ Detect pulse rising edge
   - ✓ Limited resolution
3. **Pulses modulation and matched filter**
   - ✓ **Correlation with local replica of signal**
   - ✓ **Simple solution**
   - ✓ **Optimal filter to maximize SNR**

False alarm and detection probabilities



False alarm probability: P_{fa}
Detection probability: P_d

$$P_{fa}(\beta) = P(X > \beta | H_0)$$

$$P_d(\beta) = P(X > \beta | H_1)$$

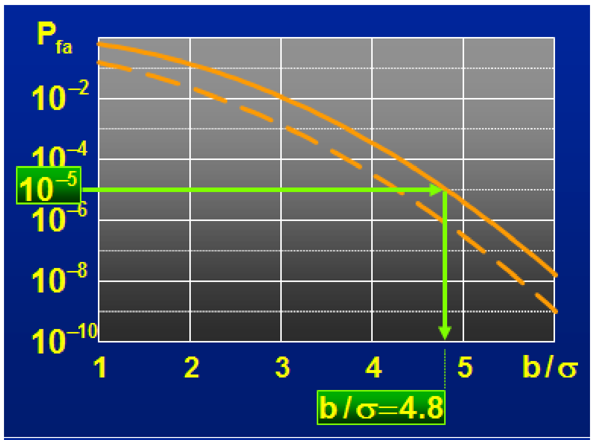
$$P_{md}(\beta) = 1 - P_d(\beta)$$

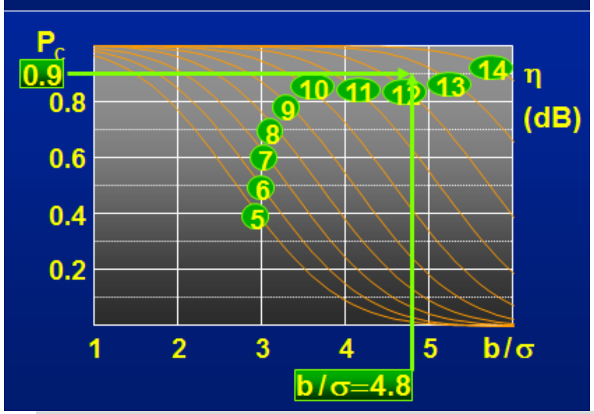
Where:

H_0 is the hypothesis of signal absence
H_1 is the hypothesis of signal presence
is a determined acquisition threshold
X is the generic random variable related to the search space

Link Budget

- Classical link budget cannot be used: no data rate, because no data are transmitted
- Radar approach, based on false alarm and detection probabilities





✓ Given a target false alarm probability, b/σ is derived:

$$P_{fa} = 10^{-5} \qquad \frac{b}{\sigma} = 4.8$$

✓ Given a target detection probability, η is derived:

$$P_d = 0.9 \qquad \eta = 12.5 \text{ dB}$$

✓ The minimum transmit power is computed:

$$\boxed{P_T > \frac{\eta(4\pi D)^2 K T_{op}}{G_{TX} G_{RX} \lambda^2 T}}$$

✓ D: distance
✓ K: Boltzmann constant
✓ G_T: TX antenna gain
✓ G_R: RX antenna gain
✓ λ: wavelength
✓ T: pulse width

Orbits

Orbits

- Extensive flight time over south pole
  - Eccentric
  - Longitude of ascending node = 0°
  - Argument of perilune = 90°

- Stability
  - Frozen orbits $e = \left(1 - \frac{5}{3}\cos^2 i\right)^{1/2}$
    - Null variations of eccentricity and argument of perilune due to compensation of J_2, J_3, J_5
  - Librating orbits
    - Quasi-periodic variations of eccentricity and argument of perilune

- Coverage and precision
  - 6 satellites for first constellation (100 m positioning error)
  - 12 satellites for final constellation (50 m positioning error)

Frozen Orbits

Eccentricity: $$\frac{\partial e}{\partial t} = \frac{15}{8}\frac{n_3^{\ 2}}{n} e\left(1-e^2\right)^{\frac{1}{2}} sin^2 i \, sin\, 2\omega$$

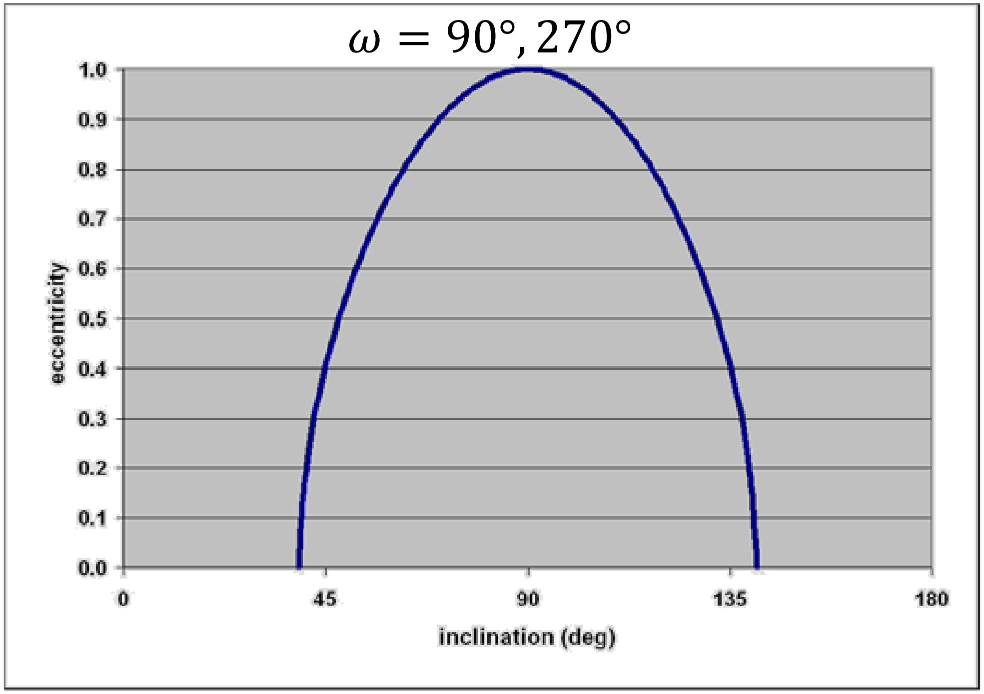
Arg. of Periapsis: $$\frac{\partial \omega}{\partial t} = \frac{3}{16}\frac{n_3^{\ 2}}{n}\frac{1}{\left(1-e^2\right)^{\frac{1}{2}}}\left[\left(3+2e^2+5\cos 2i\right)+5\left(1-2e^2-\cos 2i\right)\cos 2\omega\right]$$

$$\frac{d\omega}{dt} = 0 \qquad \frac{de}{dt} = 0$$

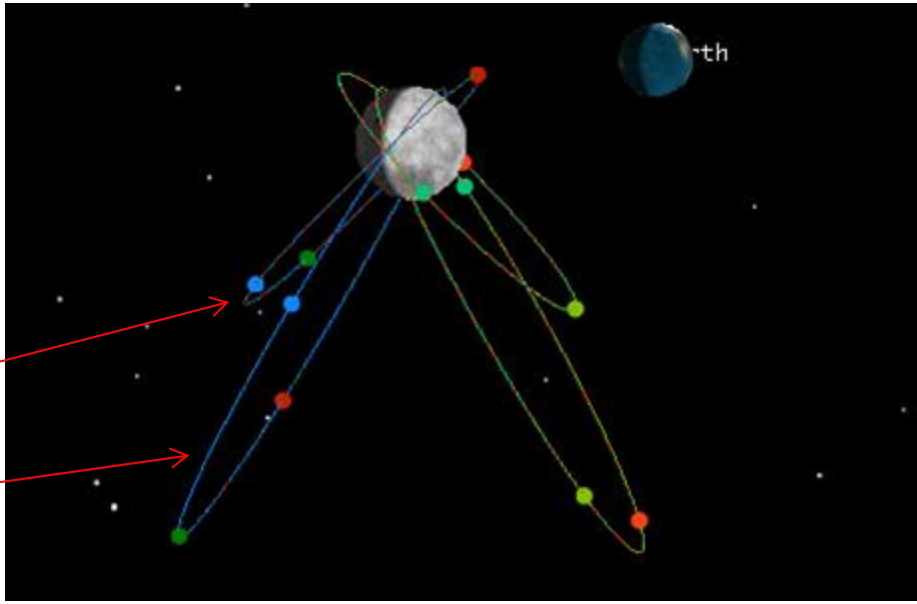
Excluding trivial cases, i.e. equatorial (i=0°,180°), circular (e=0), escape (e=1), we can find solutions for the eccentricity rate when $\omega = 90°, 270°$. Now, the perilune rate is null for

$$\omega = 90°, 270° \Rightarrow e = \left(1 - \frac{5}{3}\cos^2 i\right)^{1/2}$$

Orbits



$$e = \left(1 - \frac{5}{3}\cos^2 i\right)^{1/2}$$



e = 0.4028, i = 45°, 135°

e = 0.7638, i = 60°, 120°

Ground Tests

Ground Tests

Test	Q/A	Description
Static load tests	Q	The flight structure is held in a rigid test stand representing the launch vehicle adapter and subjected to limit, yield and ultimate loads through hydraulic jacks.
Spin test	Q	Required to simulate the spinning motion after launcher separation.
Sinusoidal vibration test	Q, A	A shaker subjects the spacecraft to a “sweep” of sinusoidal frequencies according to the values prescribed in the launch vehicle manual.
Shock test	Q	Shocks like shroud jettison and spacecraft separation are simulated.
Physical properties test	Q, A	Mass, center of gravity and moments of inertia are determined, needed for attitude control design and for the setup of other mechanical tests.
Thermal vacuum tests	Q, A	Verification of the electrical functionality in the vacuum of space under the extreme temperatures to which the spacecraft will be subjected.
Thermal balance test	Q	Simulation of the mission thermal environment (solar radiation, albedo, internal dissipation), performed in a vacuum solar simulation chamber.
Electromagnetic compatibility test	Q, A	The possibility of electromagnetic interference from external sources is analyzed. This test is run in an anechoic room.